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Assignment 04

Q1 Represent Beta function.

Ans Beta function / Euler's Ist Integral

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \quad ; \quad m,n>0$$

Properties

① $B(m,n) = B(n,m)$

② $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

③ $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

Q2 Represent Gamma Formula.

Ans Gamma function, denoted as Γn

$$\Gamma n = \int_0^\infty e^{-x}\cdot x^{n-1}\,dx \quad ; \quad n>0$$

Properties

① $\Gamma(n+1) = n\,\Gamma n$

② $\Gamma n = (n-1)!$

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Q3 Generate Riemann Integration of $\int_0^b 3x \, dx$.

Sol $\int_0^b F(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} F(x_i).\Delta x$

$\Delta x = \frac{b-a}{n} = \frac{b-0}{n} = \frac{b}{n}$

$\Rightarrow \quad \Delta x = \frac{b}{n}$

$\Rightarrow \quad x_i = a + \Delta x . i$

$x_i = 0 + \frac{b}{n} i = \frac{ib}{n}$

$x_i = \frac{ib}{n}$

Now,

$\int_0^b 3x \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} 3F(x_i).\Delta x$

$\Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} 3 \frac{ib}{n} . \frac{b}{n} \Rightarrow \lim_{n\to\infty} \frac{3b^2}{n^2} [1+2+\cdots+n]$

$\Rightarrow \lim_{n\to\infty} \frac{3b^2}{n^{\cancel{2}}} \left[\frac{\cancel{n}(n+1)}{2}\right]$

$\Rightarrow \lim_{n\to\infty} \frac{3b^2}{2} \left[1 + \cancel{\frac{1}{n}}^{\,0}\right]$

$\Rightarrow \boxed{\frac{3b^2}{2}}$ Ans

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Q4 Generate Riemann Integration of $\int_0^a x^2\,dx$.

Sol $\int_0^a F(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} F(x_i)\cdot \Delta x$

$\Rightarrow \Delta x = \frac{a-0}{n} = \frac{a}{n} \Rightarrow \Delta x = \frac{a}{n}$

$\Rightarrow x_i = 0 + \Delta x \cdot i = \frac{a}{n} i \Rightarrow x_i = \frac{ia}{n}$

Now, $\int_0^a x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{ia}{n}\right)^2 \left(\frac{a}{n}\right)$

$\Rightarrow \lim_{n\to\infty} \frac{a^2}{n^2}\cdot\frac{a}{n} \sum_{i=1}^{n} i^2$

$\Rightarrow \lim_{n\to\infty} \frac{a^3}{n^{\cancel{3}2}} \left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$

$\Rightarrow \lim_{n\to\infty} \frac{a^3}{6\cancel{n^2}} \cancel{n}\left(1+\frac{1}{n}\right) \cancel{n}\left(2+\frac{1}{n}\right)$

$\Rightarrow \lim_{n\to\infty} \frac{a^3}{6} \left(1+\cancel{\frac{1}{n}}^{0}\right)\left(2+\cancel{\frac{1}{n}}^{0}\right)$

$\Rightarrow \boxed{\frac{a^3}{3}}$ Ans

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Q5 Evaluate $\iint (x+y)\,dx\,dy$ where the region of integration is $3x+2y \leq 1$ in the positive quadrant.

Sol Given,

$$\iint (x+y)\,dx\,dy$$

Solve the conditions :

$$3x+2y \leq 1$$

when $y=0$, $x = \frac{1}{3}$

and for y in terms of x is $y = \frac{1-3x}{2}$

So,
integral bounds for $y=0$ to $y = \frac{1}{2} - \frac{3x}{2}$

and for x, the bounds from 0 to $\frac{1}{3}$

Now,
$$\int_0^{1/3}\int_0^{\frac{1}{2}-\frac{3x}{2}} (x+y)\,dx\,dy$$

$$\Rightarrow \int_0^{1/3}\left[xy + \frac{y^2}{2}\right]_0^{\frac{1}{2}-\frac{3x}{2}} dx$$

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$$\Rightarrow \int_0^{1/3} \left[x\left(\frac{1}{2} - \frac{3x}{2}\right) + \frac{\left(\frac{1}{2} - \frac{3x}{2}\right)^2}{2} - 0 \right] dx$$

$$\Rightarrow \int_0^{1/3} \left[\frac{x}{2} - \frac{3x^2}{2} + \frac{1}{2}\left(\frac{1}{4} + \frac{9x^2}{4} - \frac{3x}{2} \right) \right] dx$$

$$\Rightarrow \int_0^{1/3} \left(\frac{x}{2} - \frac{3x^2}{2} + \frac{1}{8} + \frac{9x^2}{8} - \frac{3x}{4} \right) dx$$

$$\Rightarrow \left[\frac{x^2}{4} - \frac{3x^3}{6} + \frac{x}{8} + \frac{9x^3}{24} - \frac{3x^2}{8} \right]_0^{1/3}$$

$$\Rightarrow \frac{1}{4}\left(\frac{1}{3}\right)^2 - \frac{1}{2}\left(\frac{1}{3}\right)^3 + \frac{1}{8}\left(\frac{1}{3}\right) + \frac{3}{8}\left(\frac{1}{3}\right)^3 - \frac{3}{8}\left(\frac{1}{3}\right)^2$$

$$\Rightarrow \frac{1}{36} - \frac{1}{54} + \cancel{\frac{1}{24}} + \frac{1}{72} - \cancel{\frac{1}{24}}$$

$$\Rightarrow \frac{3}{72} - \frac{1}{54}$$

$$\Rightarrow \boxed{\frac{5}{216}}$$ Ans